Summer school on Fourier analytic and probabilistic methods in geometric functional analysis and convexity.
List of short courses (3-4 Lectures each).

- **Alexander Barvinok:** Some applications of functional analysis to discrete geometry: counting integer points.
  Abstract: I plan to discuss problems related to counting non-negative integer matrices with prescribed row and column sums and arising probabilistic phenomena. More generally, I would like to discuss counting integer points in polytopes, algorithms, and a discrete version of the Brunn-Minkowski inequality. Methods of higher-dimensional geometry, log-concavity, and the Prekopa-Leindler inequality, in particular, as well as various questions relating to matrix scaling and concentration inequalities turn out to be relevant. The presentation is intended to be self-contained.

- **Piotr Indyk:** Graph expansion and explicit constructions in high-dimensional geometry. Abstract: I will survey recent results that use expander graphs and their relatives to obtain explicit (as opposed to probabilistic) constructions of a variety of geometric structures. In particular, I will show a few explicit constructions of "large" near-Euclidean sections of the $\ell_1^n$ unit ball, which provide an explicit version of Dvoretzky theorem for $\ell_1$. I will also describe various applications of those constructions to problems in computer science and digital signal processing.

- **Fedor Nazarov:** Turan’s lemma, Erdos lemniscate length problem, The first self-intersection for a random walk on a finite graph and more.
  Abstract: My current plan is to use the first 3 lectures to discuss three unrelated topics: Turan’s lemma, Erdos lemniscate length problem and the problem about the expected time of the first self-intersection for a random walk on a finite graph. I hope to spend the fourth lecture talking about my favorite unsolved problems.

- **Krzysztof Oleszkiewicz:** Introduction to harmonic analysis and probability on a discrete cube.
  Abstract: Several basic notions and tools for the analysis on a discrete cube will be introduced, including Walsh-Fourier system and hypercontractivity. Some applications to problems coming from various branches of mathematics will follow.

- **Gideon Schechtman:** Euclidean (and other nice) sections of convex bodies.
  Abstract: I’ll survey the proof of the celebrated theorem of Dvoretzky stating that symmetric convex bodies have central sections, of dimension tending to infinity with the dimension of the body, which are arbitrarily close to Euclidean balls. I’ll also talk on some relatively recent developments regarding the evaluation of the dimensions of these Euclidean balls. I may also speak on the existence of other high dimensional nice sections of convex bodies.

- **Rolf Schneider:** The use of spherical harmonics in convex geometry. Abstract: These introductory lectures aim at describing the role of elementary harmonic analysis in convex geometry. First we explain the basic facts about spherical harmonics and their relation to representations of the rotation group. Then we present selected applications. They belong to two categories. In the first group, the problems lead to functional equations which are invariant under rotations. In the second group, series expansions into spherical harmonics can be used to obtain geometric inequalities and stability results. Some of the examples are very old, some quite recent.
• **Mikhail Sodin:** "Random Complex Zeroes."

Here is a tentative plan of my lectures:

– **Lecture 1.** Zeroes of Gaussian analytic functions (Edelman-Kostlan formula, Calabi’s rigidity, Offord-type estimate).

– **Lecture 2.** Fluctuations of zeroes of GEF (Gaussian Entire Functions, fluctuations of linear statistics, asymptotic normality).

– **Lecture 3.** A story of one piece-wise linear function (Large and moderate deviations, Gaussian perturbations of the lattice, Jancovici-Lebowitz-Manificat law, hole probability).

– **Lecture 4.** Transportation of the Lebesgue measure to zeroes of GEF (Transportation distance, uniform spreading and vector fields, gradient transportation).

• **Santosh S. Vempala:** Algorithmic Convex Geometry. We consider the efficient solution of basic problems in high-dimensional spaces, namely optimization, integration, rounding, centroid computation, learning etc. In the case of convex bodies, all of these problems are reducible to the problem of sampling efficiently. Most of the presentation will describe algorithms and proof techniques establishing upper and lower bounds on the complexity of sampling arbitrary convex bodies. On the way, we will encounter geometric random walks, new isoperimetric inequalities and various useful properties of logconcavity.